Classical & Quantum Waves

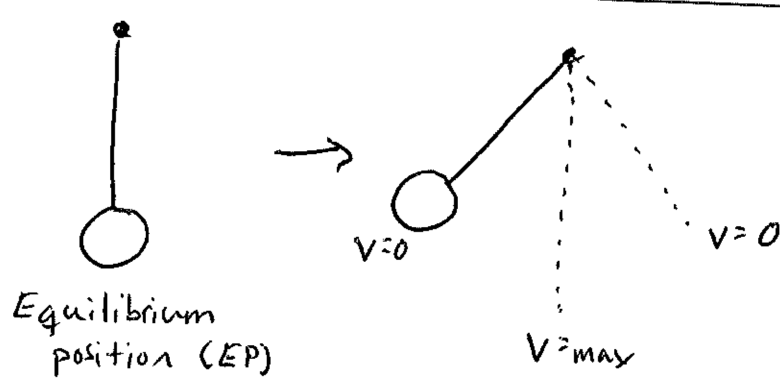
- Oscillations, vibrations & waves are all around us and at the heart of many branches of physics & engineering
- (See intro slides)

1) Simple harmonic motion / oscillator

- Everything in the physical world can vibrate or oscillate
  - swinging pendulum
  - guitar string
  - electron in an atom
  - photons in an optical cavity
  - electrons in an RLC circuit

1.1 Characteristics of simple harmonic oscillator (SHO)

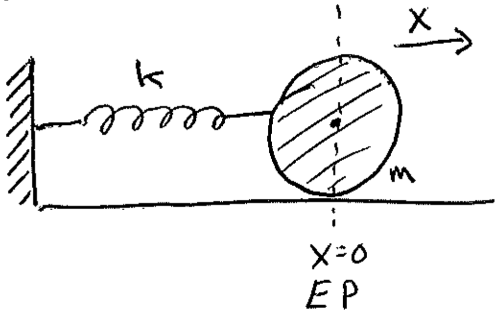
e.g., simple pendulum



For all classical SHOs, there is:

- periodic motion
- Equilibrium position (EP)
- restoring force toward EP
- inertia causing overshoot
- Flow of energy between potential & kinetic energy

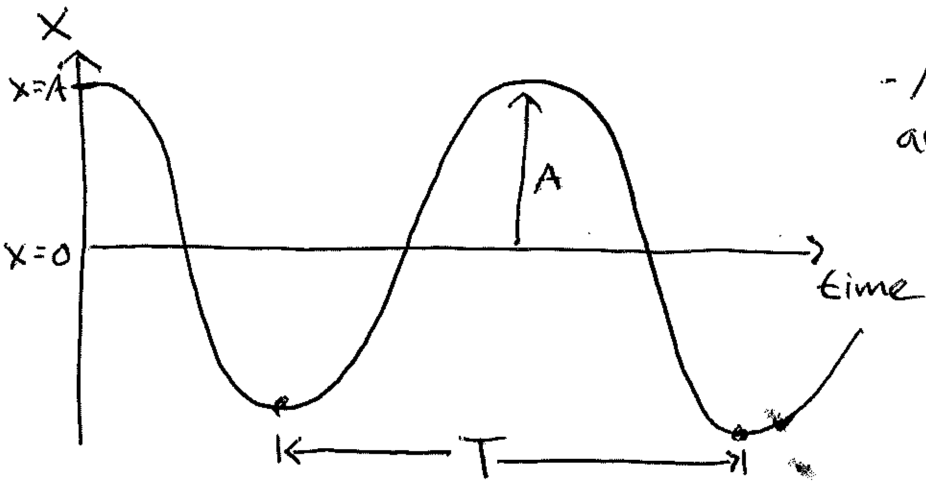
1.2 Mass on a spring



Assume idealized situation as a "model"
- frictionless surface
- spring is massless

→ First find solutions to this simple, ideal case.
Then add complications reflecting reality later.

- Pull the mass away from EP ~~and release~~ by amount A, then release



- Mass will start to oscillate around its EP, at $x=0$
- Maximum displacement is called Amplitude A
- initial condition was $x=A$ at $t=0$

- The time taken for one complete cycle of oscillation is called the Period T
- # of cycles per unit time is called Frequency ν
- Period and frequency are inverses of each other:

$$\boxed{\nu = \frac{1}{T}}$$

unit: 1 Hz ("Hertz") $\equiv$ 1 cycle/second $\equiv$ 1/sec

- Force produced by idealized spring is described by Hooke's law:

$$\boxed{F = -kx}$$ with spring constant k

→ magnitude of force directly proportional to displacement
So the more you pull on the spring, the bigger the force

→ Force always directed towards EP
(why there is a minus sign)

- System must obey Newton's 2nd law, that any force gives rise to an acceleration:

$$F = ma = -kx \qquad \text{where } a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

$$\rightarrow m\frac{d^2x}{dt^2} = -kx$$

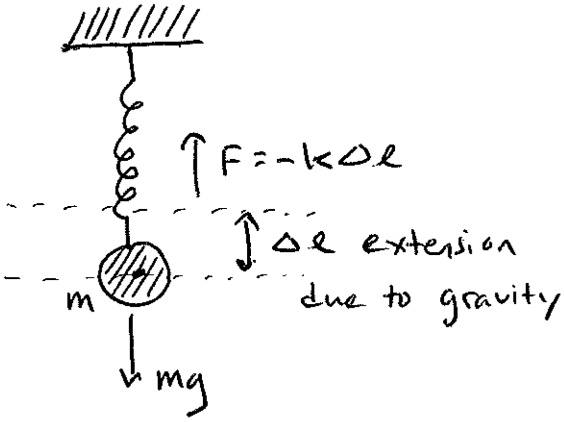
$$\rightarrow \boxed{\frac{d^2x}{dt^2} = -\omega^2 x} \qquad \text{where } \omega^2 \equiv \frac{k}{m}$$

(why we define like this clear later)

- Linear second-order differential equation. Solve to get solutions $x(t)$

- All SHOs have eq. of this form!

Case 2: vertical spring/mass system



- When spring/mass is hung vertically, gravity pulls down on the mass and stretches the spring by an extra amount $\Delta\ell$

- Equilibrium is attained when the spring stretches enough such that the Hookes' force compensates gravity force

$$\rightarrow \quad mg - k\Delta\ell = 0 \quad \rightarrow \quad k\Delta\ell = mg$$

- Resultant force if displaced:

$$F = m\frac{d^2x}{dt^2} = mg - k(\Delta\ell + x) = \underbrace{mg - k\Delta\ell}_{=0} - kx$$

$$\rightarrow \quad m\frac{d^2x}{dt^2} = -kx$$

→ Eq. stays identical, only the EP changed, shifting from $x = 0$ to $x = \Delta\ell$